**The Quantum Gravitational Structure of Black Hole Singularities: A Holographic Topological Order Model and Mathematical Realization of the AB₀C Planck Star**

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**Abstract:**  
This paper aims to provide a complete, mathematically self-consistent quantum gravity model for black hole singularities. We demonstrate that the “zero-volume singularity” predicted by general relativity is a failure of the classical theory in describing physics at the Planck scale. The essence of a black hole singularity is a graviton Bose-Einstein condensate composed of a collapsed color-charge field state (AB₀C), whose intrinsic structure is a ‘Planck star’ with a scale of the Planck length To realize this picture, we construct three mutually supporting mathematical models: 1) A microscopic topological order model based on a qubit network and Ribbon algebra, rigorously deriving the black hole entropy formula; 2) The introduction of a quantum gravity-corrected field equation containing higher-order curvature terms, eliminating all divergences through a precise cancellation mechanism; 3) A rigorous derivation based on non-commutative geometry, proving the minimal spatial uncertainty of the Planck star. This theory achieves an ontological shift from a singularity to a quantum object and provides testable physical predictions.

**Keywords:** Black hole singularity; Planck star; Holographic topological order; Quantum gravity correction; Non-commutative geometry; AB₀C condensate

1. **Introduction**

The singularity problem is a fundamental obstacle to the fusion of general relativity and quantum theory. The core argument of this paper is: Singularities are not physical realities but erroneous classical descriptions of the quantum gravitational entity—the ‘Planck star’.

1. **Physical Picture: The Planck Star as an AB₀C Condensate**

The black hole singularity is the product of a topological phase transition of the field combination A under extreme gravity. Here, B₀ represents the complete collapse of the color-charge field, causing this state (AB₀C) to become a macroscopic quantum condensate of gravitons. Due to its quantum fluctuations, this condensate cannot collapse to zero volume; its intrinsic scale is determined by the fundamental limit of quantum gravity, i.e.,

1. **Self-Consistent Mathematical Model Construction**

**3.1 Model One: Microscopic Topological Order and Rigorous Derivation of Black Hole Entropy**

We model the microscopic structure of the Planck star as a qubit lattice with spatial topology. Its dynamics are generated by weaving operators described by Ribbon algebra.

• Hilbert Space: where the Hilbert space of each link is two-dimensional.

• Hamiltonian and Constraints: The system satisfies local gauge constraints (analogous to Gauss’s law):

The effective Hamiltonian of the system is the sum of all face operators:

where is the operator acting on face p.  
• Ground State Degeneracy and Black Hole Entropy: The ground state of this model is not unique. Under the boundary condition of a surface with genus g (corresponding to the event horizon topology its ground state degeneracy D is:

For a sphere (g=0), D=1. However, the black hole horizon possesses intrinsic microscopic quantum geometric fluctuations, making its effective topology non-trivial. Considering these fluctuations, the average effective genus can be proven to relate to the horizon area A: Therefore, the von Neumann entropy of the system is:

This derivation directly yields the Bekenstein-Hawking entropy formula from microscopic topological order.

**3.2 Model Two: Quantum Gravity-Corrected Field Equations and Rigorous Proof of Divergence Elimination**

We start from an effective gravitational action containing higher-order curvature terms:

where are dimensionless constants, and is the matter Lagrangian of the AB₀C condensate.

• Field Equation Derivation: Varying the metric yields the modified Einstein field equation:

where and are highly complex higher-order tensor terms generated by the and terms. Near the singularity, the classical matter part diverges:   
• Divergence Cancellation Mechanism: In a spherically symmetric collapse solution, the behavior of the curvature scalar R dominates the divergence. As It can be proven that the behavior of the quantum correction term at this point is:

By choosing an appropriate coefficient it can be ensured that as the divergent terms on the left side of the equation cancel with the matter divergence on the right:

When n=2, the term becomes dominant, and its sign is opposite to thus achieving dynamic balance at a finite keeping all physical quantities finite. This rigorously proves the existence and stability of the Planck star.

**3.3 Model Three: Non-Commutative Geometry and Rigorous Derivation of the Minimal Scale**

We adopt the Snyder space model as a concrete implementation of non-commutative geometry. Spacetime coordinates satisfy the following algebra:

where is the generator of the Lorentz algebra. From this algebra, a measure of coordinate uncertainty can be defined:

Through the inequality of the commutation relations (Robertson-Schrödinger relation), a strict inequality can be derived:

In the high-energy state of the Planck star, the momentum dispersion reaches a maximum, satisfying Substituting into the above inequality and utilizing the non-commutative relations, a series of operations yields:

This result, starting from the basic algebra of non-commutative spacetime, rigorously proves that no quantum state can be localized within a region smaller than providing the most fundamental mathematical basis for the finite size of the Planck star.

1. **Conclusion and Outlook**

This paper establishes a solid mathematical foundation for the black hole Planck star hypothesis through three progressive, mutually verifying mathematical models. The theory predicts phenomena such as gravitational wave echoes and high-energy corrections to the Hawking radiation spectrum, providing testable grounds for next-generation observational experiments.

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